

# A Review of Signal Parameter Estimation Techniques

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In signal analysis, the signals to be detected usually contain unknown parameters such as amplitude, time delay, phase, and frequency; these parameters must be estimated prior to the signal detection. The techniques used to estimate these signal parameters can be broadly classified into two main categories known as parametric and non-parametric methods. This paper presents a review of these signal parameter estimation techniques.

## Introduction

Signal parameter estimation, and hence detection, problems are concerned with the analysis of received signals to determine the absence or presence of a signal of interest; the extraction of information in these signals as well as the signal classification [1]. These are problems of significance in applications such as seismic exploration, speech recognition, cellular mobile communication, biomedical engineering, radar and sonar signal processing. The signals to be detected often contain unknown parameters, such as amplitude, time delay, phase, and frequency; these parameters must be estimated before any signal detection. To estimate these parameters, a number of methods can be applied. Generally, signal parameter estimation techniques can be classified into two main categories; namely parametric and non-parametric approaches [2-4]. A review of some of these signal parameter estimation methods is presented in this paper.

## Parametric and Non-Parametric Signal Parameter Estimation Techniques

Non-parametric techniques are Fourier-based methods of providing spectral estimates where no prior model is assumed, in the sense that no assumptions are made concerning the physical process that generated a given data. They are also known as the classical methods of spectral estimation. Although this approach of signal parameter estimation is computationally efficient, it however has limited frequency resolution. These methods also suffer from spectral leakage effects that often mask weak signals. Prominent conclusions from these non-parametric techniques are that there is always a compromise in the bias-variance trade-off because both of these errors cannot be minimised simultaneously [2-5].

Parametric-based methods can however be used to extract high-resolution estimates, especially in applications where short data records are available due to transient phenomena, provided the signal structure is known. These techniques are also known as model-based methods of spectral estimation, where a generating model with known functional form is assumed. The parameters in the assumed model are then estimated, and a signal's spectral characteristics of interest derived from the estimated model. Therefore, the estimated spectral characteristics are only as good as the underlying model. Examples of these parametric-based techniques include the autoregressive (AR) process model (comprising the Yule-Walker [4, 6] and least squares methods [4]), the moving average (MA) process model, as well as the combined autoregressive moving average (ARMA) process model [2-4]. The autoregressive process model, such as the Prony algorithm, is the simplest of the parametric-based techniques. The Prony algorithm, which models sampled data as a linear combination of exponentials, is a technique that can be used for identifying the frequencies, amplitudes, and phases of a signal. Although the Prony algorithm has the ability to resolve rays much closer than the Fourier-based limit, it however has a tendency to yield biased estimates. An improved version of the Prony algorithm, named "singular value decomposition followed by prony-type root recovery," is

often referred to as singular value decomposition prony (SVDP).

A further example, of the parametric-based methods, is the space-alternating generalized expectation-maximization (SAGE) algorithm [7]. The SAGE algorithm is a low-complexity generalisation of the expectation-maximization (EM) algorithm [8]. The EM algorithm is an iterative procedure used to compute a maximum likelihood estimate when an observed data is regarded as incomplete [9]. The SAGE algorithm breaks down a multi-dimensional optimisation process, necessary to compute the estimates of the parameters of a wave, into several separate, low-dimensional maximisation procedures, which are performed sequentially; thereby reducing the computational cost. Furthermore, this algorithm overpowers the resolution limitation inherent in the Fourier-based methods. However, the SAGE algorithm depends on the assumption that a finite known number of waves characterised by their propagation delay, complex amplitude, and azimuthal incidence direction are impinging in the neighbourhood of a receiver. Under-estimating the number of impinging waves can result in poor resolution, while over-estimation can give rise to spurious components in the parameter estimates.

Another class of these parametric-based estimation methods is the subspace-based technique. This method, also known as super-resolution or high-resolution techniques, generate frequency component estimates of a given signal based on the decomposition of an observation vector space into two subspaces; one associated with the signal and another associated with the noise [2, 9]. Each noise vector is assumed to be uncorrelated with the signal vectors and among other noise vectors. Then the functions corresponding to the vectors in the signal or noise subspaces can be used to create frequency estimators which, when plotted, indicate sharp peaks at the frequency locations of interest. Pisarenko harmonic decomposition (PHD) algorithm [2-4] was the first of these methods, which consequently spurred many improved methods such as the multiple signal classification (MUSIC) algorithm [10]. The MUSIC algorithm was initially used for azimuth estimation. The algorithm was later applied to time delay estimation. The MUSIC algorithm gives better resolution than the autoregressive or Prony methods. Although MUSIC was the first of the high-resolution algorithms to accurately exploit the underlying data model of signals that are buried in noise, this algorithm however has several limitations. For example, a complete knowledge of the array manifold is needed, and the search over parameter space is computationally expensive. A polynomial-rooting version of the MUSIC technique, known as "root-MUSIC," is known to have similar asymptotic properties as the conventional MUSIC algorithm. Moreover, this root-MUSIC technique is plagued by spurious roots which cause problems in identifying the actual roots corresponding to the true signals.

Other examples of these subspace-based techniques include the minimum norm method [11] and estimation of signal parameters by rotational invariance techniques (ESPRIT) method [12]. The ESPRIT is an extension of the MUSIC algorithm. ESPRIT uses two or more arrays that bear a translation invariance relationship with respect to each other and then exploits the underlying rotational invariance among the signal subspaces to solve a generalised eigenvalue equation. This algorithm has two variants; the original ESPRIT, and a total least squares (TLS) version of the original technique. These two variants of ESPRIT are known to give similar asymptotic estimation accuracy. However, the TLS version has lower bias in the frequency estimates. ESPRIT exhibits significantly low computational complexity over the MUSIC algorithm and produces estimates that are asymptotically unbiased.

A summary of the advantages and disadvantages of these signal parameter estimation methods is presented in Table I.

## Advantages and Disadvantages of Some Signal Parameter Estimation Techniques

*Table I. Summary of the Advantages and Disadvantages of Some Signal Parameter Estimation Techniques [13].*

Method	Advantages	Disadvantages
Yule-Walker Algorithm [2, 3, 14].	(1) Computationally efficient. (2) Produces better resolution than Fourier-based methods.	(1) The model order needs to be specified in advance of the analysis. (2) Performs relatively poorly for short data records.
Least Squares Method [2-4, 14].	(1) Has superior performance than the Yule-Walker algorithm. (2) Yield statistically stable spectral estimates.	(1) The model order needs to be specified in advance of the analysis. (2) The resolution for signals with low signal-to-noise ratios (SNRs) is comparable to that obtained from Fourier-based methods.
Pisarenko Harmonic Decomposition [2-4, 9, 15].	Computationally efficient.	(1) The performance is poor at low SNRs. (2) The model order needs to be specified in advance of the analysis.

Method	Advantages	Disadvantages
Extended Prony Algorithm [2, 3, 14].	(1) Parameter estimates are less biased than those obtained from the Pisarenko method. (2) Can resolve delays to better than half the Fourier limit.	(1) The model order needs to be specified in advance of the analysis. (2) Resolution degrades at low SNR scenarios.
MUSIC Algorithm [10].	(1) Has better resolution than Prony-based algorithm. (2) Yields asymptotically unbiased parameter estimates.	(1) High computational burden. (2) The model order needs to be specified in advance of the analysis. (3) Fails to resolve closely spaced signals at low SNRs.
Minimum Norm [16, 17].	(1) Has lower computational cost, and better resolution, than the MUSIC algorithm. (2) Optimises the separation of the spurious roots in root-MUSIC.	Exhibit spurious peaks, and merging of spectral peaks, at low SNR values.
TLS-ESPRIT [9].	(1) Produces less biased estimates. (2) More accurate than conventional ESPRIT. (3) Manifests superior performance than the Pisarenko and minimum norm methods.	(1) Requires an accurate estimate of the number of signals. (2) Has higher computational cost than conventional ESPRIT.
SAGE Algorithm [7].	(1) Has lower computational cost than the MUSIC algorithm. (2) Yields better resolution than Fourier-based approaches.	The number of impinging waves needs to be specified in advance of the analysis.
Independent Component Analysis [18].	(1) Lower sensitivity to SNRs, number-of-paths, and bandwidth, when compared with the MUSIC algorithm. (2) Has lower computational cost than the MUSIC algorithm.	Requires proper selection of a cost function.
gold-MUSIC Algorithm [19].	(1) Low sensitivity to different SNR conditions. (2) Has quick convergence.	gold-MUSIC and conventional MUSIC algorithm follows the same steps, until the isolation of the noise eigenvectors, which requires an accurate estimate of the number of signals.

## Conclusions

In this paper, a review of signal parameter estimation techniques has been presented. It was shown that while the classical (or Fourier-based) methods of signal parameter estimation are computationally efficient, they however have limited frequency resolution. Moreover, parametric (or model-based) signal parameter estimation techniques can be useful in extracting high-resolution estimates. Examples of these model-based parameter estimation methods, reviewed in this paper, include the autoregressive process models, the space-alternating generalized expectation-maximization algorithm, and subspace-based techniques. ■

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